**S1 Text**

The equation of motion for a viscously damped multi-degree-of-freedom system can be expressed as:

|  |  |
| --- | --- |
|  | (1s) |

where [m], [c], and [k] are the mass, damping, and stiffness matrices, is the displacement vector, and is the force vector. The central difference formulas for the velocity and acceleration vectors at time ti = i ∆t ( ) are given by:

|  |  |
| --- | --- |
| = ()-1 ( | (2s) |
| = ()-2 ( | (3s) |

Thus the equation of motion at time ti can be written as:

|  |  |
| --- | --- |
| [M] ()-2 ( + [C] ()-1 ( + [K] = | (4s) |

Where , , , , and ti = i ∆t. Eq.4s can be rearranged to obtain:

|  |  |
| --- | --- |
| (()-2 [M] + ()-1 [C]) = – ([K] - 2()-2 [M])  – (()-2 [M] – (2)-1 [C]) | (5s) |

Thus Eq. (5s) gives the solution vector once and are known. Since Eq. (5s) is to be used for i = 1,2,….,n, the evaluation of requires and . Thus a special starting procedure is needed to find . For this, Eqs. (1s)- (3s) are evaluated at i=0 to obtain:

|  |  |
| --- | --- |
|  | (6s) |
| = ()-1 ( | (7s) |
| = ()-2 ( | (8s) |

Eq. 6s gives the initial acceleration vector as:

|  |  |
| --- | --- |
|  | (9s) |

And Eq.7s gives the displacement vector at t1 as:

|  |  |
| --- | --- |
|  | (10s) |

Substituting Eq. 10s for , Eq. 8s yields:

|  |  |
| --- | --- |
| = 2 ()-2 ( | (11s) |